Personal Finance: Another Perspective

Time Value of Money 2: Inflation, Real Returns, Annuities, and Amortized Loans

2019/07/08

Objectives

A. Understand the impact of inflation on investing

B. Understand how to calculate real returns

C. Know how to solve problems relating to Annuities, Future Value of an annuity, and Present Value of an annuity

D. Know how to solve problems relating to amortized loans

A. Understand the Impact of Inflation on Investing

- What is inflation?
  - Inflation is the increase in the volume of money and credit relative to available goods and services resulting in a continuing rise in the general price level
    - In other words, your money can buy less goods and services than it could before.
    - To compensate, you must pay more money (for the same amount of goods)

Inflation (continued)

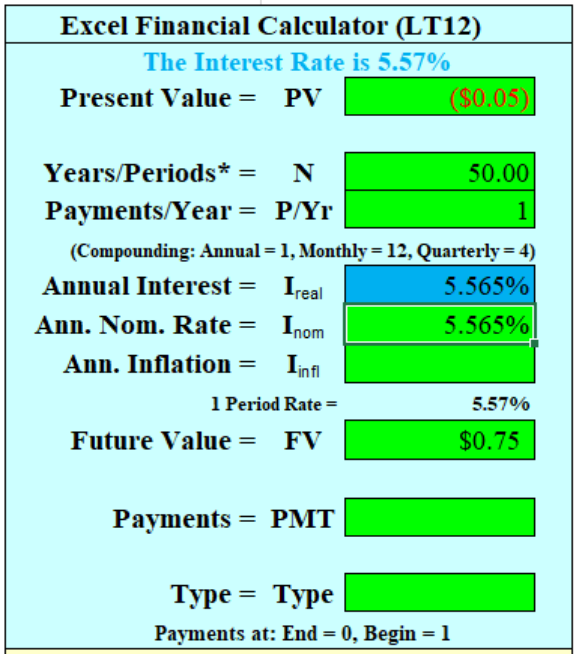
- What is the impact of inflation on investing?
  - Inflation has a negative impact on your investments
    - You have the same amount of money but your money can buy less
      - Inflation forces us to save more, because our dollars will be worth less in the future.

Investment Problem #1
Inflation

- 40 years ago a five stick pack of gum was 5 cents a pack. Today it is 75 cents per pack.
  - Assuming no change in the size of the gum, at what rate has inflation increased over each of those 50 years?

Answer #1: Inflation

- Clear registers, set your calculator to 1 payment year, and set your payments to end mode, i.e. end of the period payments.
  - -.05 = PV .75 = FV 50 = N and solve for I
    - I% = 5.56% per year
  - Inflation on gum has averaged 5.56% per year for the last 50 years!



Investment Problem #2: The Future Value of a Wedding (inflation)

- I have six wonderful daughters (and one son). Assume the average wedding costs $23,000 this year.
  - Assuming 4% inflation, what will it cost in 15 years for all six weddings for my six daughters?

Answer #2:
Future Costs of Weddings and Inflation

- Clear registers, set to end of period, 1 payment per year
  - -23,000 = PV, 15 = N, 4 = I
    Solve for FV ?
  - FV = $41,422 per wedding
- Total Cost = $41,422 x 6 = ?
  - Total costs = $248,530
    - I need to save now
      Inflation will raise
      my costs by 80%!!!!

Excel Financial Calculator (LT12)

The Future Value is $41,421.70

Present Value =	PV	($23,000.00)
Years/Periods* =	N	15.00
Payments/Year =	P/Yr	1

(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)

Annual Interest =	I_{real}	4.000%
Ann. Nom. Rate =	I_{nom}	4.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		4.00%
Future Value =	FV	$41,421.70

B. Understand the Concept of Real Returns and how to calculate them

- Real Returns are your return on investments after the impact of inflation
  - Inflation has a negative impact on your investments, i.e. your money will buy less
- To keep your buying power constant (or your real returns constant) you must actually earn more money in nominal terms to maintain your purchasing power
  - The formula for Real Returns is: $\frac{(1 + \text{return})}{(1 + \text{inflation})} - 1$

Real Return Methodology

- While some have argued that it is OK to just subtract inflation (π) from your nominal return (r_{nom}).
  - However, this overstates your real return (r_{real}).
- The correct formula links $(1+r_{real}) * (1+\pi) = (1 + r_{nom})$
  - Multiplied out and simplified the answer is:

    $r_{real} + \pi + r_{real}\,\pi = r_{nom}$
  - Assuming the cross term $r_{real}\,\pi$ is small, the formula condenses to: $r_{real} + \pi = r_{nom}$ or $r_{real} = r_{nom} - \pi$
- The correct method is to divide both sides by $(1+\pi)$ and subtract 1 to give: $r_{real} = [(1 + r_{nom})/ (1+\pi)] - 1$

Investment Problem #3: Real Returns (returns after inflation)

- Paul just graduated from college and landed a "real" job that pays $35,000 per year.
  - A. What nominal rate will he need to earn in the future to maintain a 2% real return assuming inflation averages 1.96%?
  - B. What will his salary be in 10 years in nominal terms?
  - C. What will his salary be in 10 years in real terms?

Answer #3:
Real Returns

A. What nominal rate is needed to maintain a 2% real return?

- Real Return = (1 + nominal return)/(1 + inflation) – 1
- To make a real return of 2%, he will need to make a nominal return of 4% assuming inflation of 1.96%

  1 + .02 = (1 + x%)/(1.0196) Solve for x?

  His nominal return must be 4%

Answer #3:

Real Returns (continued)

B. What will his salary be in 10 years in nominal terms?

- Nominal return: -$35,000 = PV,
  4 = I%, 10 = N, P/Yr = 1,
  solve for FV ?
- FV = $51,809

Excel Financial Calculator (LT12)		
The Future Value is $51,808.55		
Present Value =	PV	($35,000.00)
Years/Periods* =	N	10.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	4.000%
Ann. Nom. Rate =	I_{nom}	4.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		4.00%
Future Value =	FV	$51,808.55

C. What will his salary be in 10 years in real terms?

- Real Return: -35,000 = PV
  2 = I%, P/Yr = 1, 10 = N
  Solve for FV ?
- FV = $42,665

Excel Financial Calculator (LT12)		
The Future Value is $42,664.80		
Present Value =	PV	($35,000.00)
Years/Periods* =	N	10.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	2.000%
Ann. Nom. Rate =	I_{nom}	2.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		2.00%
Future Value =	FV	$42,664.80

C. Know How to Solve Problems Relating to Annuities

- What is an Annuity?
  - A series of equal dollar payments coming at the end of each time period for a specified number of time periods, generally months or years.
- What is a Compound Annuity?
  - An investment that involved depositing an equal sum of money at the end of each year for a certain number of year and allowing it to grow
- Can you find the Present and Future Value of each of these types?
  - Yes!

Solving Problems Relating to Annuities

Problem:

- You want to determine the value of a series of equal dollar payments either currently or at some point in the future

Problem Statement:

- What will be the future (present) value of a series of annual (monthly) payments x years in the future if my interest rate is y%?

Key Information Needed:

- Amounts, years in the future and the interest rate

Investment Problem #4: Annuities

- When you retire at age 60, you plan to have $750,000 saved. Assuming an interest rate of 7% and that you will need payments for 30 years (you expect to die at age 90), how much could you receive each year for your $750,000?

Answer #4: Annuities

- Assuming you need payments for 30 years and interest rates are 7%, you can receive:
  - -750,000= PV, 30 = N,

    7 = I%, P/Yr = 1

    Solve for Payment?
  - PV of Annuity = $60,440
- Start saving for retirement the month you graduate. It will make a big difference on what you will be able to retire with!

Excel Financial Calculator (LT12)		
The Payment is $60,439.80		
Present Value =	PV	($750,000.00)
Years/Periods* =	N	30.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	7.000%
Ann. Nom. Rate =	I_{nom}	7.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		7.00%
Future Value =	FV	
Payments =	PMT	$60,439.80

Investment Problem #5: Compound Annuities

- You are looking to buy a new Polaris 700 quad or four wheeler to clean the snow from your driveway (ha!). Instead of borrowing the $7,000 for the quad, you want to save for it each ***month***. How much will you need to save each month, assuming you can get 7% on your investments and you want to buy the quad in 50 months? How much if you can get 7% and want it in 24 months?

Answer #5: Compound Annuities

- 50 Months:
  - Clear memories, set payments to 12 (monthly), set to end mode
  - -7,000 = FV, 50/12 = N, 7 = I%, P/Yr = 12, Solve for PMT = ?
  - Note that for N, it needs to be an annual number, or 50/12
  - You will need to save $120.98 each month to have it in 50 months

Excel Financial Calculator (LT12)

The Payment is $1.28

Present Value =	PV	
Years/Periods* =	N	4.17
Payments/Year =	P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	7.000%
Ann. Nom. Rate =	I_{nom}	7.000%
Ann. Inflation =	I_{infl}	
12 Period Rate =		0.58%
Future Value =	FV	($7,000.00)
Payments =	PMT	$120.98
Type =	Type	

Payments at: End = 0, Begin = 1

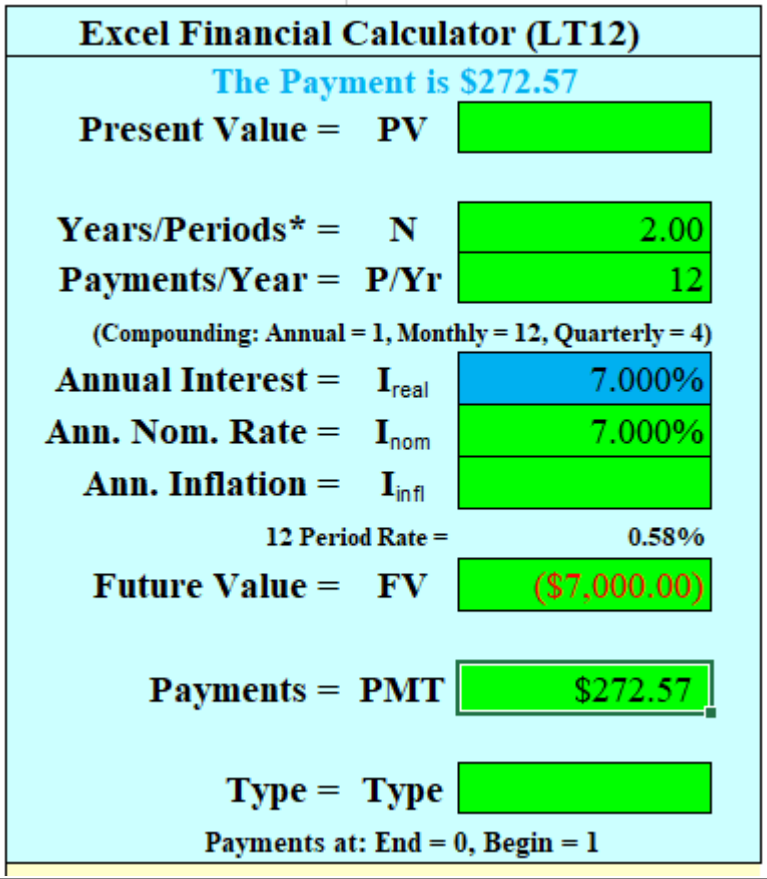
Answer #5: Compound Annuities

- 24 months:
  - Clear memories, set payments to 12 (monthly), set to end mode

    -7,000 = FV, 24/12 = N,

    P/Yr = 12, 7 = I%,

    Solve for PMT = ?
  - You will need to save $272.57 each month to have it in 24 months



Investment Problem #6: Present Value of an Annuity

- Two people wish to buy your house. The first offers you $200,000 today, while the second offers you 25 annual payments of $14,200 each.
  - What is the present value of each offer, assuming a 5% discount rate?
  - Who would you sell it to?

Answer #6:
Present Value of Annuities

A. What is the Present Value of each offer:

Offer 1: Present value = $200,000

Offer 2: Clear memories, set 1 payment year and end of period

- -14,200 = PMT, 25 = N, 5 = I%, Solve for PV?
- Present Value of Offer 2 = $200,134

B. Which is the better offer?

Offer 1 and Offer 2 are roughly the same

- Take either one (one is more certain)
- Know how to evaluate different cash flows!

Excel Financial Calculator (LT12)

The Present Value is $4,193.30

Present Value =	PV	$200,134.01
Years/Periods* =	N	25.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	5.000%
Ann. Nom. Rate =	I_{nom}	5.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		5.00%
Future Value =	FV	
Payments =	PMT	($14,200.00)
Type =	Type	
Payments at: End = 0, Begin = 1		

Investment Problem #7: FV of an Annuity

- Josephine, age 22, started working full time and plans to deposit $3,000 annually into an IRA earning 6%. How much will he have (FV) in:
  - 20 years?
  - 30 Years?
  - 40 Years?
- If she increased her investment return to 8%, how much would he have after each of the three time periods?

Answer #7:
Present Value of an Annuity

- 6% interest, $3,000 PMT
  - 20 Years = $110,357
  - 30 Years = $237,175
  - 40 Years = $464,286
- 8% Interest
  - 20 Years = $137,286 ($26,929 more)
  - 30 Years = $339,850 ($10,675 more)
  - 40 Years = $777,170 ($312,884 more)
- Your rate of return and time make a big difference!

Excel Financial Calculator (LT12)

The Future Value is $110,356.77

Present Value =	PV	
Years/Periods* =	N	20.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	6.000%
Ann. Nom. Rate =	I_{nom}	6.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		6.00%
Future Value =	FV	$110,356.77
Payments =	PMT	($3,000.00)
Type =	Type	

Payments at: End = 0, Begin = 1

Excel Financial Calculator (LT12)

The Future Value is $110,356.77

Present Value =	PV	
Years/Periods* =	N	20.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	8.000%
Ann. Nom. Rate =	I_{nom}	8.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		8.00%
Future Value =	FV	$137,285.89
Payments =	PMT	($3,000.00)
Type =	Type	

Payments at: End = 0, Begin = 1

D. Know how to solve problems relating to Amortized Loans

- What is an Amortized Loan?
  - A loan paid off in equal installments, both principle and interest
    - With an amortized loan the interest payment declines as your outstanding principal declines; therefore, with each payment you will be paying an increasing amount towards the principal of the loan.
      - Examples -- car loans or home mortgages

Solving Amortized Loan Problems

- Problem:
  - You want to determine either the payment, time or interest rate necessary for a loan
- Problem Statement:
  - If I want to borrow z amount for x years at y% interest, what will be my p or payment?
- Key Information Needed:
  - Borrowed amount, years, interest rates, and payments

Investment Problem #8: Buying a Car (With Four "Easy" Payments)

•What are the annual payments to borrow $36,000 at 15% interest for a new Suburban SUV with four annual payments? How much interest will you have paid for the SUV?

Answer #8:
Buying that SUV

A. Clear memories, set payments to annual and to end mode

PV = -36,000, N = 4, I = 15, P/Yr =1

Solve for PMT = ?

PMT = $12,609.55

B. Interest Paid = 12,609.55 * 4 =

$50,438.2 – $36,000 = $14,438.21

(that is the cost of another car!)

Buy the car, but not on credit.

Save for it!

Excel Financial Calculator (LT12)

The Payment is $12,609.55

Present Value =	PV	($36,000.00)
Years/Periods* =	N	4.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	15.000%
Ann. Nom. Rate =	I_{nom}	15.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		15.00%
Future Value =	FV	
Payments =	PMT	$12,609.55
Type =	Type	

Payments at: End = 0, Begin = 1

Investment Problem #9: Buying a House

- Calculate the ***monthly*** payments for a $250,000 mortgage loaned in each of the following ways:
  - 30 year fixed at 4.5%
  - 15 year fixed at 3.75%
  - 20 year fixed at 4.125%
  - What are the total payments on each loan and which is the best option for a homeowner who, after reviewing his budget, can afford $1,550 per month?
- Note: some calculators require you to set the payments to 12 (for monthly payments) and also require you to take the interest rate and divide by 12. Others just require you to set the payments to 12. Determine what your calculator requires before solving problems requiring monthly data!

Answer #9: The House

- 30 year at 4.5%
  - Clear memories, set to monthly payments, and set to end mode

    -250,000 = PV, 30 = N,

    4.5% = I, P/Yr = 12

    Solve for PMT?
  - PMT = $1,267

Excel Financial Calculator (LT12)		
The Payment is $1,266.71		
Present Value =	PV	($250,000.00)
Years/Periods* =	N	30.00
Payments/Year =	P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	4.500%
Ann. Nom. Rate =	I_{nom}	4.500%
Ann. Inflation =	I_{infl}	
12 Period Rate =		0.38%
Future Value =	FV	
Payments =	PMT	$1,266.71
Type =	Type	
Payments at: End = 0, Begin = 1		

Answer #9: The House

- 15 year at 3.75%
  - Clear memories, set to monthly payments, and end mode
  - -250,000 = PV 15 = N
  - 3.75% = I, P/Yr = 12
  - Solve for PMT?
  - PMT = $1,818

Excel Financial Calculator (LT12)

The Payment is $1,482.22

Present Value =	PV	($250,000.00)
Years/Periods* =	N	15.00
Payments/Year =	P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	3.750%
Ann. Nom. Rate =	I_{nom}	3.750%
Ann. Inflation =	I_{infl}	
12 Period Rate =		0.31%
Future Value =	FV	
Payments =	PMT	$1,818.06
Type =	Type	

Payments at: End = 0, Begin = 1

Answer #9:
The House (continued)

- 20 year at 4.125%
  - Clear memories, set to monthly payments, and end mode
  - -250,000 = PV 20 = N
  - 4.125% = I, P/Yr = 12
  - Solve for PMT?
  - PMT = $1,531
- The safest option would be the 20-year fixed rate mortgage at 4.125 percent. It allows the homebuyer to pay off the home in ten less years than the 30 year loan and to save $77,250 in interest costs versus the 30 year loan.

Investment Problem #10: Becoming a Millionaire

- Your buddy thinks that to become a millionaire is totally beyond his earning abilities. You, the financial wizard that you are, plan to show her otherwise. Assuming your friend is 25 years old, and will retire at age 65, and assuming an 6% interest rate, how much will she have to save each month to reach her goal of becoming a millionaire when she retires?
- How much each month if she earns 8% on her investments?

Answer #10: the Millionaire

- At 6% interest:
  - Clear your memory and set payments to monthly.
    FV = 1,000,000 N = 40,

    P/Yr = 12, I = 6%,

    Solve for Payment (PMT)
  - PMT = $502.14

Excel Financial Calculator (LT12)		
The Payment is $502.14		
Present Value =	PV	
Years/Periods* =	N	40.00
Payments/Year =	P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	6.000%
Ann. Nom. Rate =	I_{nom}	6.000%
Ann. Inflation =	I_{infl}	
12 Period Rate =		0.50%
Future Value =	FV	($1,000,000.00)
Payments =	PMT	$502.14

Answer #10: the Millionaire

- At 8% interest:
  - Clear your memory and set payments to monthly.
    FV = 1,000,000 N = 40,

    P/Yr = 12, I = 8%,

    Solve for Payment (PMT)
  - PMT = $286.45

Excel Financial Calculator (LT12)		
The Payment is $286.45		
Present Value =	PV	
Years/Periods* =	N	40.00
Payments/Year =	P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	8.000%
Ann. Nom. Rate =	I_{nom}	8.000%
Ann. Inflation =	I_{infl}	
12 Period Rate =		0.67%
Future Value =	FV	($1,000,000.00)
Payments =	PMT	$286.45
Type =	Type	
Payments at: End = 0, Begin = 1		

- Its not that hard to become a millionaire if you will invest a specific amount every month!

Questions

- Are you comfortable with solving amortized loan problems?

Review of Objectives

A. Do you understand the impact of inflation on investing?

B. Do you know how to calculate real returns?

C. Can you solve problems relating to Annuities, Future Value of an annuity, and Present Value of an annuity?

D. Do you know how to solve problems relating to amortized loans?

Case Study #1

Data

- Lee is thirty-five years old and makes a $4,000 payment *every year* into a Roth Individual Retirement Account (IRA) (this is an annuity) for thirty years.

Calculations

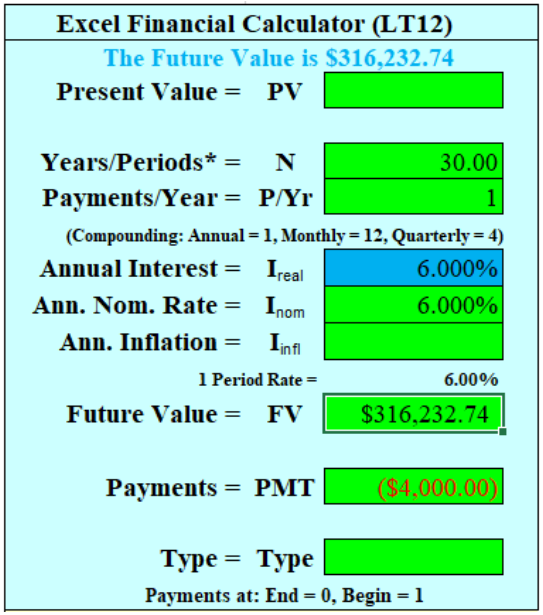
- Assuming the discount, or interest, rate Lee will earn is 6%, what will be the value of his Roth IRA investment when he retires in *30 years* (this is future value)?

  Note: The math formula is a bit tricky. The formula is:

  FVn = Payment * (FVIFAi,n)

Case Study #1 Answers

- There are two ways for Lee to solve the problem. Using the math formula, the problem is solved:
- $FV_{n,i}$ = Payment * $[(1 + i)^n - 1] / i$ = FV = \$4,000 * [(1.06)30 –1]/.06 = \$316,232.74
- Using a financial calculator, clear the calculator's memory and solve the problem this way:
- 1 = P/Y (payments per year)
- 4,000 = PMT (payment)
- 6 = I (interest rate)
- 30 = N (number of years)
- Solve for FV = \$316,232.74



Case Study #2

Data

- Janice will make a *yearly* $2,000 payment (this is an annuity) for forty years into a traditional IRA account.

Calculations

- Given that the discount, or interest, rate is 6 percent, what is the *current value* (that is, the present value) of Janice's investment in today's dollars? The math formula is:

  $PV_{n,i}$ = Payment * ($PVIFA_{n,i}$)

Case Study #2 Answer

- Using the math formula, the calculation is:
- $PV_{n,i}$ = Payment * [1–(1/(1 + i)^n)]/i = PV = 2,000 * [1–(1/(1.06)40] /.06 = $30,092.59
- Using the financial calculator, the calculation is:
- Clear memories and use the following:
- 1 = P/Y (payments per year)
- 2,000 = PMT (payment)
- 6 = I (interest rate)
- 40 = N (number of years)
- Solve for PV = $30,092.59

Excel Financial Calculator (LT12)

The Present Value is $30,092.59

Present Value =	PV	$30,092.59
Years/Periods* =	N	40.00
Payments/Year =	P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)		
Annual Interest =	I_{real}	6.000%
Ann. Nom. Rate =	I_{nom}	6.000%
Ann. Inflation =	I_{infl}	
1 Period Rate =		6.00%
Future Value =	FV	
Payments =	PMT	($2,000.00)
Type =	Type	

Payments at: End = 0, Begin = 1

Case Study #3

Data

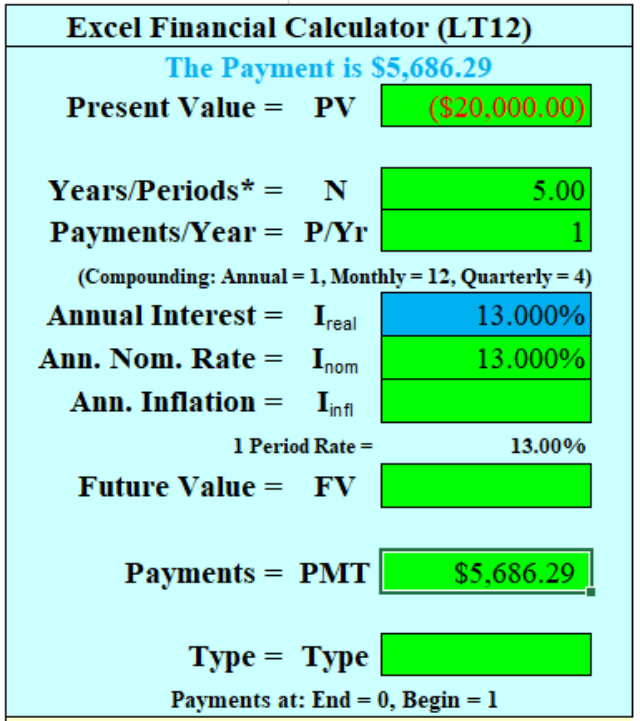
- Brady wants to borrow $20,000 dollars for a new car at 13 percent interest.

Calculations

- He wants to repay the loan in five *annual* payments (this is an annuity). How much will he have to pay *each year* (this indicates present value)? The math formula is the formula that was used in the previous problem, i.e.:
- $PV_n = Payment * (PVIFA_{i,n})$

Case Study #3 Answer

- Using the math formula, put Brady's borrowed amount into the equation and solve for your payment. $PV_{i,n}$ = Payment * [1–(1/(1 + i)^n)]/i = PV = $20,000 = Payment * [1–(1/(1.13)5] /.13 = $5,686.29 per year.
- Using a financial calculator, clear the calculator's memory and use the following:
- 1 = P/Y (payments per year)
- 20000 = PV
- 13 = I (interest rate)
- 5 = N (number of years)
- Solve for PMT = $5,686.29



Case Study #4

Data

- Kaili has reviewed the impact of inflation in the late 1970s. She reviewed one of her parent's investments during that time period. Inflation was 20 percent and her parent's investment made a 30 percent return.

Calculations

- What was her parent's real return on this investment during that period?

Case Study #4 Answers

- The traditional (and incorrect) method for calculating real returns is:
  - Nominal return – inflation = real return. This formula would give her a real return of 10%.
    - 30% – 20% = 10%
  - The correct method is: (1 + nominal return)/(1 + inflation) – 1 = real return or (1.30/1.20)–1 = 8.33%
- In this example, the traditional method overstates the real return by 20 percent ((10%/8.33%) –1). Be very careful of inflation, especially high inflation!

Case Study #4 Answers

- The correct formula for calculating inflation is very similar to a single period geometric return. The formula is:
  - (1 + real return) * (1 + inflation) = (1 + nominal return).
  - If you multiply out the left side [(1 + real return) * (1 + inflation)], you get [1 + inflation + real return + a cross-term which is inflation * real return]. If you assume this cross-term is small or zero, then the formula condenses to:
  - Real return + inflation = nominal return which is an approximation.

Case Study #4 Answers

- The correct way to solve for the real return is to begin with the above formula:
  - (1 + real return) * (1 + inflation) =
  - (1 + nominal return)
  - Divide both sides by (1 + inflation) and subtract 1 from both sides and you get the
  - formula discussed earlier:
    - Real Return = $\frac{(1 + \text{nominal return})}{(1 + \text{inflation})}$