

10. Time Value of Money 2: Understanding Inflation, Annuities, and Amortized Loans

Introduction

This chapter continues the discussion on the time value of money. In this chapter, you will learn how inflation impacts your investments; you will also learn how to calculate real returns after inflation as well as annuities and payments on amortized loans.

Objectives

Once you have completed this chapter, you should be able to do the following:

1. Explain how inflation impacts your investments.
2. Understand how to calculate real returns (returns after inflation).
3. Solve problems related to annuities.
4. Solve problems related to amortized loans.

Explain How Inflation Impacts Your Investments

Inflation is an increase in the volume of available money in relation to the volume of available goods and services; inflation results in a continual rise in the price of various goods and services. In other words, because of increased inflation, your money can buy fewer goods and services today than it could have bought in the past.

Inflation negatively impacts your investments. Although the amount of money you are saving now will be the same amount in the future, you will not be able to buy as much with that money in the future (the purchasing power of your money erodes). Inflation makes it necessary to save more because your currency will be worth less in the future.

Problem 1: Inflation

Forty years ago, gum cost five cents a pack. Today it costs 99 cents a pack. Assume that the increase in the price of gum is completely related to inflation and not to other factors. At what rate has inflation increased over the last 40 years?

Before solving this problem, clear your calculator's memory, and set your calculator to one annual payment. Then input the following information to solve this problem:

$$PV = -\$0.05 \text{ (the price of gum forty years ago)}$$

FV = \$0.75 (the price of gum today)
 N = 40 (The cost has increased every year for forty years.)
 I = ?

The formula is: $((FV/PF)^{(1/N))}-1$

On average, the inflation rate has been 5.56 percent each year for the last 40 years. So, the average price of gum has increased by 5.56 percent each year for the last 40 years.

Excel Financial Calculator (LT12)	
The Interest Rate is 5.57%	
Present Value = PV	(\$0.05)
Years/Periods* = N	50.00
Payments/Year = P/Yr	1
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I _{real}	5.565%
Ann. Nom. Rate = I _{nom}	5.565%
Ann. Inflation = I _{infl}	
1 Period Rate =	5.57%
Future Value = FV	\$0.75
Payments = PMT	
Type = Type	
<small>Payments at: End = 0, Begin = 1</small>	

Problem 2: Inflation—The Future Value of a Wedding

I have six daughters and one son. It is estimated that an average wedding cost \$23,000. Assuming four-percent inflation, what would it cost me to pay for all six of my daughters' weddings in 15 years? (Hopefully not all six weddings will take place in the same year.)

Before you begin, clear your calculator's memory and set your calculator to one annual payment. Input the following information to solve for the cost of a single wedding in 15 years:

PV = \$23,000 (Assume that on average a wedding still costs \$23,000.)
 N = 15 (The cost will increase every year for 15 years.)
 I = 4 (The inflation rate is four percent.)
 FV = ?

The formula is: $PV*((1+I)^{(N)})$.

Excel Financial Calculator (LT12)	
The Future Value is \$41,421.70	
Present Value = PV	(\$23,000.00)
Years/Periods* = N	15.00
Payments/Year = P/Yr	1
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I _{real}	4.000%
Ann. Nom. Rate = I _{nom}	4.000%
Ann. Inflation = I _{infl}	
1 Period Rate =	4.00%
Future Value = FV	\$41,421.70

In 15 years, the value of a single wedding will be \$41,422. This means six weddings will cost \$248,530. Inflation will raise my costs by 80 percent $((\$41,422 / 23,000) - 1)$ over the next 15 years, so I need to plan now.

Understand How to Calculate Real Returns

A real return is the rate of return you receive after the impact of inflation. As discussed earlier, inflation has a negative impact on your investments because your money will buy less in the future. For example, 40 years ago a gallon of gas cost 25 cents per gallon; currently, gas costs \$4.00 per gallon. While the gas itself hasn't changed (much), the price has increased. To keep your real return constant (in other words, to maintain your buying power), you must actually earn more money in nominal (not inflation adjusted) terms.

Excel Financial Calculator (LT12)	
The Future Value is \$51,808.55	
Present Value = PV	(\$35,000.00)
Years/Periods* = N	10.00
Payments/Year = P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)	
Annual Interest = I _{real}	4.000%
Ann. Nom. Rate = I _{nom}	4.000%
Ann. Inflation = I _{infl}	
1 Period Rate = 4.00%	
Future Value = FV	\$51,808.55

Traditionally, investors have calculated the real return (r_r) as simply the nominal return (r_n), or the return you receive, minus the inflation rate (π). This method is incorrect. It is preferable to use the following formula:

$$(1 + \text{nominal return } (r_n)) = (1 + \text{real return } (r_r)) * (1 + \text{inflation } (\pi))$$

To solve for the real return, divide both sides of the equation by $(1 + \text{inflation } (\pi))$. Once you've divided, the equation looks like this:

$$(1 + \text{nominal return } (r_n)) / (1 + \text{inflation } (\pi)) = (1 + \text{real return } (r_r))$$

Then, subtract one from both sides and reverse the equation to get the following:

$$\text{Real return } (r_r) = [(1 + \text{nominal return } (r_n)) / (1 + \text{inflation } (\pi))] - 1$$

Problem 3: Real Return (i.e., the Return after Inflation)

Paul just graduated from college and landed a job that pays \$35,000 per year. Assume that inflation averages 1.96 percent per year.

A. What nominal rate will Paul need to earn in the future to maintain a 2-percent real return rate?

B. In nominal terms, what will Paul's salary be in 10 years? Assume that his salary keeps up with inflation and that inflation averages the same 1.96 percent per year.

a. To determine the nominal rate of return, remember the formula for real return: $r_r = ((1 + r_n) / (1 + \pi)) - 1$. Now plug in the values you know: $0.02 = (1 + x) / (1 + 0.0196) - 1$. Solving for x results in a nominal return of 4.00 percent. Thus, Paul's nominal return must be 4.00 percent in the future to maintain a real return of 2 percent. The formula for the nominal rate of return is $NR = (1 + RR) * (1 + I) - 1$.

b. To maintain his current purchasing power 10 years from now, Paul will have to make \$51,809 in real terms.

This problem is very similar to the Future Value we have already discussed. Use the following values to solve this problem:

PV = -\$35,000 (This is Paul's current salary.)

I = 2 (Interest is replaced by inflation.)

$N = 10$ (This is the number of years in the future.)
Solve for $FV = ?$

Excel Financial Calculator (LT12)	
The Future Value is \$42,664.80	
Present Value = PV	(\$35,000.00)
Years/Periods* = N	10.00
Payments/Year = P/Yr	1
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I_{real}	2.000%
Ann. Nom. Rate = I_{nom}	2.000%
Ann. Inflation = I_{infl}	
1 Period Rate =	2.00%
Future Value = FV	\$42,664.80

The formula is $FV = PV * (1+I)^N$.

Understand How to Solve Problems Related to Annuities

An annuity is a series of equal payments that a financial institution makes to an investor; these payments are made at the end of each period (usually a month or a year) for a specific number of years. To set up an annuity, an investor and a financial institution (for example, an insurance company) sign a contract in which the investor agrees to transfer a specific amount of money to the financial institution, and the financial institution, in turn, agrees to pay the investor a set amount of money at the end of each period for a specific number of years.

To determine the set amount of each equal payment for a certain investment, you must know the amount of the investment (PV), the interest rate (I), and the number of years the annuity will last (N).

Problem 4: Annuities

{XE “Annuities” } When you retire at age 60, you have \$750,000 in your retirement fund. The financial institution you have invested your money with will pay you an interest rate of 7 percent. Assuming you live to age 90, you will need to receive payments for 30 years after you retire. How much can you expect to receive each year for your \$750,000 investment with a 7 percent interest rate?

To solve this problem, input the following information into your financial calculator:

Set $-\$750,000$ as your present value (PV). Your present value is negative because it is considered an outflow. You pay this amount to the financial institution, and the financial institution pays you back with annual payments.

Set 30 as the number of years (N).

Set 7 percent as your interest rate (I). Remember that you may need to convert this percentage to the decimal 0.07 in some calculators.

Now solve for the payment (PMT). The present value of this annuity is \$60,439.80. This means you should receive 30 annual payments of \$60,439.80 each.

Without a financial calculator, solving this problem is a bit trickier. The formula is as follows:

$$PMT = PV_{N,I} / ((1 - (1 / (1 + I)^N)) / I)$$

$$PMT = \$750,000 / ((1 - (1 / (1.07)^{30})) / 0.07) = \$60,439.80.$$

Excel Financial Calculator (LT12)	
The Payment is \$60,439.80	
Present Value = PV	(\$750,000.00)
Years/Periods* = N	30.00
Payments/Year = P/Yr	1
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)	
Annual Interest = I _{real}	7.000%
Ann. Nom. Rate = I _{nom}	7.000%
Ann. Inflation = I _{infl}	
1 Period Rate =	7.00%
Future Value = FV	
Payments = PMT	\$60,439.80

The key is to start saving for retirement as soon as you can. Starting to save early will make a big difference in what you are able to retire with.

Problem 5: Compound Annuities

{XE “Compound Annuities” } With a compound annuity, you deposit a set sum of money into an investment vehicle at the end of each year; you deposit this amount for a specific number of years and allow that money to grow.

Suppose you are looking to buy a new four-wheeler to remove snow from your driveway. Instead of borrowing the \$7,000 you would need to pay for the four-wheeler, you want to save for the purchase. You need to ask yourself two questions:

- A. How much will I need to save each month if I want to buy the four-wheeler in 50 months if I can earn 7 percent interest on my investment?
- B. How much will I have to save each month if I want to buy the four-wheeler in 24 months if I can earn 7 percent interest on my investment?

Note: The method you use to calculate the monthly payments will depend on the type of financial calculator you have. Some calculators require you to set the number of payments to 12 (for monthly payments) and also divide the interest rate by 12 months. Other calculators only require you to set the number of payments to 12. Determine what your calculator requires before solving problems requiring monthly data.

Before solving for the monthly payment, follow these steps: (1) clear your calculator’s memory, (2) set your number of payments to 12 so that your calculator will calculate monthly payments instead of annual payments, and (3) make sure your calculator is operating in “end mode,” since the payments are received at the end of each period.

To solve the first question, input the following information:

$$\begin{aligned} FV &= -\$7,000 \\ N &= 50/12 \\ I &= 7 \\ PMT &= ? \end{aligned}$$

If you earn 7 percent interest on your investment, you will need to save \$120.98 each month to save \$7,000 in 50 months. If you do not have a financial calculator, use the following to solve this problem:

$$\text{The formula is } \text{PMT} = \text{FV}_{N,I} / (((1 + (I / 12))^N) - 1) / (I / 12))$$

$$\text{PMT} = \$7,000 / (((1 + (0.07 / 12))^50) - 1) / (0.07 / 12)) = \$120.98$$

Excel Financial Calculator (LT12)	
The Payment is \$1.28	
Present Value = PV	
Years/Periods* = N	4.17
Payments/Year = P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)	
Annual Interest = I _{real}	7.000%
Ann. Nom. Rate = I _{nom}	7.000%
Ann. Inflation = I _{infl}	
12 Period Rate =	0.58%
Future Value = FV	(\$7,000.00)
Payments = PMT	\$120.98
Type = Type	
Payments at: End = 0, Begin = 1	

To solve the second question, input the following information:

$$\text{FV} = -\$7,000$$

$$\text{N} = 24$$

$$\text{I} = 7$$

$$\text{PMT} = ?$$

After solving for the payment, you will discover that you need to save \$272.57 each month to save \$7,000 in 24 months. If you do not have a financial calculator, use the following to solve this

problem:

$$\text{PMT} = \$7,000 / (((1 + (0.07 / 12))^24) - 1) / (0.07 / 12)) = \$272.57$$

Excel Financial Calculator (LT12)	
The Payment is \$272.57	
Present Value = PV	
Years/Periods* = N	2.00
Payments/Year = P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)	
Annual Interest = I _{real}	7.000%
Ann. Nom. Rate = I _{nom}	7.000%
Ann. Inflation = I _{infl}	
12 Period Rate =	0.58%
Future Value = FV	(\$7,000.00)
Payments = PMT	\$272.57
Type = Type	
Payments at: End = 0, Begin = 1	

As a general rule, it is better to save for a purchase than to borrow money for it because when you borrow you will have to pay interest instead of earning interest.

Problem 6: Present Value of Annuities

Let's try another sample problem using annuities; this time, we will be calculating the present value instead of the set payment amount.

There are two people who each want to buy your house. The first person offers you \$200,000 today, while the second person offers you 25 annual payments of \$15,000. Assume a 5 percent interest or discount rate. What is the present value of each offer? If you could take either offer, which person would you sell your house to?

1. First offer: The present value of this offer is \$200,000 because the buyer can pay you all of the money today.

2. Second offer: This offer is a little different because you will not receive all of the money today; therefore, you must calculate the present value.

To calculate the present value of the first offer using a financial calculator, clear your calculator's memory, set the number of payments to one annual payment, and make sure your calculator is set to "end mode." Then, input the following information:

$$\begin{aligned} \text{PMT} &= -\$15,000 \\ \text{N} &= 25 \\ \text{I} &= 5 \\ \text{PV} &= ? \end{aligned}$$

The present value of the second offer is \$211,409. If you do not have a financial calculator, use the following formula to solve for the present value:

$$\begin{aligned} \text{PV}_{\text{N,I}} &= \text{PMT} * (1 - (1 / (1 + \text{I})^{\text{N}})) / \text{I} \\ \text{PV}_{\text{N,I}} &= \$14,200 * [1 - (1 / (1.05)^{25})] / 0.05 = \$200,134 \end{aligned}$$

Excel Financial Calculator (LT12)	
The Present Value is \$4,193.30	
Present Value = PV	\$200,134.01
Years/Periods* = N	25.00
Payments/Year = P/Yr	1
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I _{real}	5.000%
Ann. Nom. Rate = I _{nom}	5.000%
Ann. Inflation = I _{infl}	
1 Period Rate =	5.00%
Future Value = FV	
Payments = PMT	(\$14,200.00)
Type = Type	
<small>Payments at: End = 0, Begin = 1</small>	

Which is the better offer? The second offer has a higher present value: if we can assume that you don't need the money right away and that you are willing to wait for payments and confident the buyer will pay you on schedule, you should accept the second offer. As you can see from this example, it is very important that you know how to evaluate different cash flows.

Problem 7: Future Value of Annuities

Just as it is possible to calculate the present value of an annuity, it is also possible to calculate the future value of an annuity.

Josephine, age 22, started working full time and plans to deposit \$3,000 annually into an IRA that earns 6 percent interest. How much will be in her IRA in 20 years? 30 years? 40 years?

To solve this problem, clear your calculator's memory and set the number of payments to one (for an annual payment). Set I equal to 6 and the PMT equal to \$3,000. The formula is: $\text{PMT} * (((1 + \text{I})^{\text{N}}) - 1) / \text{I}$.

$$\begin{aligned} \text{For 20 years: Set N equal to 20 and solve for FV. } & \text{FV} = \$110,357 \\ \text{For 30 years: Set N equal to 30 and solve for FV. } & \text{FV} = \$237,175 \\ \text{For 40 years: Set N equal to 40 and solve for FV. } & \text{FV} = \$464,286 \end{aligned}$$

If Josephine increased her return rate to 8 percent, how much money would she have after each of the three time periods? How does this interest rate compare to the 6 percent interest rate over time?

Do the previous problems at 8 percent interest. Begin by clearing the calculator's memory. Set I equal to 8 and the PMT equal to \$3,000.

For 20 years: Set N equal to 20 and solve for FV. FV = \$137,286 (\$26,929 more than she would earn at the 6 percent interest rate)

For 30 years: Set N equal to 30 and solve for FV. FV = \$339,850 (\$102,675 more than at the 6 percent rate)

For 40 years: Set N equal to 40 and solve for FV. FV = \$777,170 (\$312,884 more than at the 6 percent rate)

Your rate of return and the length of time you invest make a big difference when you retire.

Solve Problems Related to Amortized Loans

An amortized loan is paid off in equal installments (payments) made up of both principal and interest. With an amortized loan, the interest payments decrease as your outstanding principal decreases; therefore, with each payment a greater amount of money goes toward the principal of the loan. Examples of amortized loans include car loans and home mortgages.

To determine the amount of a payment, you must know the amount borrowed (PV), the number of periods during the life of the loan (N), and the interest rate on the loan (I).

Problem 8: Buying a Car

You take out a loan for \$36,000 to purchase a new car. If the interest rate on this loan is 15 percent, and you want to repay the loan in four annual payments, how much will each annual payment be? How much interest will you have paid for the car loan at the end of four years?

Before solving this problem, clear your calculator's memory and set your calculator to one annual payment. Input the following information into your financial calculator:

$$PV = -\$36,000$$

$$N = 4$$

$$I = 15$$

Solve for PMT = ? to get \$12,609.55.

$$\text{The formula is: } PMT = PV_{N,I} / ((1 - (1 / (1 + I)^N)) / I)$$

The amount of interest you will have paid after four years is equal to the total amount of the

payments ($\$12,609.55 * 4 = \$50,438.20$) minus the cost of your automobile ($\$36,000$); the total comes to $\$14,438.21$. That is one expensive loan! In fact, the interest alone is more than the cost of another less-expensive car. If you want to buy this car, go ahead, but don't buy it on credit—save for it!

Excel Financial Calculator (LT12)	
The Payment is \$12,609.55	
Present Value = PV	(\$36,000.00)
Years/Periods* = N	4.00
Payments/Year = P/Yr	1
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I _{real}	15.000%
Ann. Nom. Rate = I _{nom}	15.000%
Ann. Inflation = I _{infl}	
1 Period Rate =	15.00%
Future Value = FV	
Payments = PMT	\$12,609.55
Type = Type	
<small>Payments at: End = 0, Begin = 1</small>	

Problem 9: Buying a House

What are the *monthly* payments on each of the following mortgage loans? Which loan is the best option for a homeowner who can afford payments of \$1,550 per month? What is the total amount that will be paid for each loan? Assume each mortgage is \$250,000.

Loan A: 30-year loan with a fixed interest rate of 4.5 percent

Loan B: 15-year loan with a fixed interest rate of 5.75 percent

Loan C: 20-year loan with a fixed interest rate of 4.125 percent

Loan A. To determine the monthly payment for a 30-year loan with an 8.5-percent fixed interest rate, clear your calculator's memory, then set your calculator to 12 monthly payments and "end mode." Input the following to solve this equation:

$$PV = -\$250,000$$

$N = 360$ (Calculate the number of monthly periods by multiplying the length of the loan by the number of months in a year: $30 * 12 = 360$.)

$$I = 8.5/12$$

Solve for $PMT = ?$

Your monthly payment for this loan would be \$1,266.71, and the total amount of all payments would be $\$1,266.71 * 360$, or \$456,017. Interest is \$206,017.

The formula is: $PV / ((1 - (1 / (1 + (I/P)))^{(N * P)})) / (I/P)$

Loan B. For a 15-year loan at 3.75 percent interest, follow the same steps explained above. This time, input the information listed below:

$$PV = -\$250,000$$

$$N = 15 * 12 = 180$$

$$I = 3.75$$

PMT = ?

The monthly payment for this loan would be \$1,818.06, the total amount of all payments would be \$327,250 and the interest would be \$77,250.10.

Loan C. For a 20-year loan at 4.125 percent interest, the calculations are still the same. Input the following in your financial calculator:

PV = -\$250,000

N = 20 * 12 = 240

I = 4.125

Solve for PMT = ?

The monthly payment for this loan would be \$1,531.47, the total amount of all payments would be \$367,552 and the interest paid will be \$117,552.

Considering the mortgage payment the homeowner can afford, the best financial option is Loan C—the 20-year fixed-rate mortgage at 4.125 percent interest. This loan would allow the homeowner to pay off the home in 10 fewer years than if he or she had the 30-year loan and to pay \$77,250 less.

Problem 10: Becoming a Millionaire

Your friend thinks becoming millionaire is totally beyond her earning abilities. You, financial wizard that you are, plan to show her otherwise. Assuming your friend is 25 years old and will retire at age 65, and assuming a 6 percent interest rate, how much will she have to save each month to reach her goal of becoming a millionaire when she retires? How much each month if she earns 9 percent on her investments?

Clear your memory and set payments to monthly. FV = 1,000,000 N = (40 * 12) I = 6%, Solve for Payment (PMT).

PMT = \$502.14. She will need to save \$502 per month.

The formula is: $FV / (((1 + (I/P))^{(N*P)} - 1) / (I/P))$

At 89 percent interest:

Clear your memory and set payments to monthly. FV = 1,000,000, N = (40 * 12), I = 89%, Solve for Payment (PMT)

PMT = \$286.4513.62. She will need to save only \$28614 per month.

Excel Financial Calculator (LT12)	
The Payment is \$502.14	
Present Value = PV	
Years/Periods* = N	40.00
Payments/Year = P/Yr	12
(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)	
Annual Interest = I _{real}	6.000%
Ann. Nom. Rate = I _{nom}	6.000%
Ann. Inflation = I _{infl}	
12 Period Rate =	0.50%
Future Value = FV	\$1,000,000.00
Payments = PMT	\$502.14

It's not that hard to become a millionaire if you invest a specific amount every month and can earn a modest interest rate.

Summary

The major goal of this chapter was to help you better understand the time value of money. This chapter also helped you understand how inflation impacts your investments.

Real return is the rate of return you receive after the impact of inflation. As discussed earlier, inflation has a negative impact on your investments because you will not be able to buy as much with your money in the future. Traditionally, investors have calculated real returns with the approximation method by simply using the nominal return minus the inflation rate. Although the approximation method is fairly accurate, it can give incorrect answers when it is used for precise financial calculations. Because of the possibility of error, it is preferable to use the exact formula: $(1 + \text{nominal return } (r_n)) = (1 + \text{real return } (r_r)) * (1 + \text{inflation } (\pi)) = (1 + \text{nominal return } (r_n)) / (1 + \text{inflation } (\pi)) - 1$.

Inflation is an increase in the volume of available money in relation to the volume of available goods and services; inflation results in a continual rise in the price of various goods and services. Because of inflation, you can buy fewer goods and services with your money today than you could have bought in the past.

An amortized loan is paid off in equal installments (payments) that are made up of both principal and interest. With an amortized loan, the interest payments decrease as your outstanding principal decreases; therefore, with each payment, you pay a larger amount on the principal of the loan. Examples of amortized loans include car loans and home mortgages.

An annuity is a series of equal payments that a financial institution makes to an investor at the end of each period (usually a month or a year) for a specific number of years. A compound annuity is a type of investment in which a set sum of money is deposited into an investment vehicle at the end of each year for a specific number of years and allowed to grow. Annuities are important because they can help you prepare for retirement and allow you to receive a specific payment every period for a number of years.

Assignments

Financial Plan Assignments

There is no specific part of your PFP on the Language of Finance. However, it is an integral part

of your work and analysis. As you read through this chapter, think about the purpose of each new financial idea: annuities, present value of an annuity, and future value of an annuity. Also review the uses of amortized loans and the calculations that concern them. Using either your financial calculator or the Excel financial calculator from the Learning Tools section, make sure you understand how to solve problems of amortized loans and annuities, including the present and the future value of an annuity. It is also critical that you understand the impact of inflation on returns. Make sure you understand the correct method for calculating real returns (the return after the impact of inflation).

Learning Tools

The following Learning Tools may also be helpful as you prepare your Personal Financial Plan:

[Financial Calculator Tutorial](#) (LT03)

This document is a financial calculator tutorial about most of the major financial calculators. It also includes the financial formulas if you would prefer to program your own calculator.

[Excel Financial Calculator](#) (LT12)

This Excel spreadsheet is a simple financial calculator for those who prefer to use spreadsheets. This tool can perform most of the functions of a financial calculator, including present value, future value, payments, interest rates, and number of periods.

Review Materials

Terminology Review

Amortized Loan. An amortized loan is a loan paid off in equal installments (payments) made up of both principal and interest. With an amortized loan, the interest payments decrease as your outstanding principal decreases; therefore, with each payment a greater amount of money goes toward the principal of the loan.

Annuity. An annuity is a series of equal payments that a financial institution makes to an investor; these payments can be made at either the beginning or end of each period (usually a month or a year) for an individual's lifetime or for a specific number of years. To set up an annuity, an investor and a financial institution (for example, an insurance company) sign a contract in which the investor agrees to transfer a specific amount of money to the financial institution, and the financial institution, in turn, agrees to pay the investor a set amount of money at the end of each period for a specific number of years.

Compound Annuities. With a compound annuity, you deposit a set sum of money into an investment vehicle at the end of each year; you deposit this amount for a specific number of years and allow that money to grow.

Future Value of an Annuity. The future value of an annuity is the value of a set of

recurring payments at specific dates in the future. It measures how much you will have in the future given a specific return or interest rate.

Inflation. An increase in the volume of available money in relation to the volume of available goods and services. Inflation results in a continual rise in the price of various goods and services. In other words, because of increased inflation, your money can buy fewer goods and services today than it could have bought in the past.

Present Value of Annuity. The present value of an annuity is the current value of a set of recurring payments at specific dates in the future, given a specified rate of return or interest rate.

Real Returns. A real return is the rate of return you receive after the impact of inflation. Traditionally, investors have calculated the real return (r_r) as simply the nominal return (r_n), or the return you receive, minus the inflation rate (π). This method is incorrect. It is preferable to use the following formula: $(1 + \text{nominal return } (r_n)) = (1 + \text{real return } (r_r)) * (1 + \text{inflation } (\pi))$. To solve for the real return, divide both sides of the equation by $(1 + \text{inflation } (\pi))$. Once you've divided, the equation looks like this: $(1 + \text{nominal return } (r_n)) / (1 + \text{inflation } (\pi)) = (1 + \text{real return } (r_r))$. Then, subtract one from both sides and reverse the equation to get the following: $\text{Real return } (r_r) = [(1 + \text{nominal return } (r_n)) / (1 + \text{inflation } (\pi))] - 1$.

Review Questions

1. What is an annuity?
2. How do you set up an annuity?
3. What is a compound annuity?
4. What is the relationship between interest rate and present value?
5. What is inflation? How does it impact investments?

Case Studies

Case Study 1

Data

Lee is 35 years old and makes a \$4,000 payment *every year* into a Roth Individual Retirement Account (IRA) (this is an annuity) for 30 years.

Calculations

Assuming the discount, or interest, rate Lee will earn is 6 percent, what will be the value of his Roth IRA investment when he retires in 30 years (this is future value)?

Note: The formula is a bit tricky. It is:

$FV_{N,I} = \text{Payment} * (((1 + I)^N) - 1) / I$ (This is the future value of an annuity factor N,I)

Case Study 1 Answer

There are two ways for Lee to solve the problem. Using the formula, the problem is solved this way:

$$FV_{N,I} = \text{Payment} * (((1 + I)^N) - 1) / I = FV = \$4,000 * [(1.06)^{30} - 1] / .06 =$$

\$316,232.74

If you are using a financial calculator, clear the calculator's memory and solve:

1 = P/Y (payments per year)

4,000 = PMT (payment)

6 = I (interest rate)

30 = N (number of years)

Solve for FV = \$316,232.74

The screenshot shows the Excel Financial Calculator interface with the following values entered:

- Present Value = PV: [Blank]
- Years/Periods* = N: 30.00
- Payments/Year = P/Yr: 1
- (Compounding: Annual = 1, Monthly = 12, Quarterly = 4)
- Annual Interest = I_{nom}: 6.000%
- Ann. Nom. Rate = I_{nom}: 6.000%
- Ann. Inflation = I_{in}: [Blank]
- 1 Period Rate = 6.00%
- Future Value = FV: \$316,232.74
- Payments = PMT: \$4,000.00
- Type = Type: [Blank]
- Payments at: End = 0, Begin = 1

Case Study 2

Data

Janice will make a *yearly* \$2,000 payment for 40 years into a traditional IRA account.

Calculations

Given that the discount, or interest, rate is 6 percent, what is the current value of Janice's investment in today's dollars? The formula is:

$$PV_{N,I} = \text{Payment} * [1 - (1 / (1 + I)^N)] / I \text{ (the present value of an annuity factor } N,I)$$

Case Study 2 Answer

Using the formula, the calculation is:

$$PV_{N,I} = \text{Payment} * [1 - (1 / (1 + I)^N)] / I = PV = 2,000 * [1 - (1 / (1.06)^{40})] / .06 = \$30,092.59$$

Using the financial calculator:

Clear memories and use the following:

1 = P/Y

2,000 = PMT

6 = I

40 = N

Solve for PV = \$30,092.59

Excel Financial Calculator (LT12)	
The Present Value is \$30,092.59	
Present Value = PV	\$30,092.59
Years/Periods* = N	30.00
Payments/Year = P/Yr	1
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I _{EFF}	6.000%
Ann. Nom. Rate = I _{CON}	6.000%
Ann. Inflation = I _{OS}	
1 Period Rate =	6.00%
Future Value = FV	
Payments = PMT	\$2,000.00
Type = Type	
Payments at: End = 0, Begin = 1	

Case Study 3

Data

Brady wants to borrow \$20,000 dollars for a new car at 13 percent interest.

Calculations

He wants to repay the loan in five *annual* payments . How much will he have to pay *each year* (this indicates present value)? The formula is the same formula that was used in the previous problem:

$$PV_N = \text{Payment} * (PVIFA_{I,N})$$

Case Study 3 Answer

Using the formula, put Brady's borrowed amount into the equation and solve for your payment. $PV_{N,I} = \text{Payment} * [1 - (1 / (1 + I)^N)] / I = PV = 20,000 = \text{Payment} * [1 - (1 / (1.13)^5)] / .13 = \$5,686.29$ per year.

Using a financial calculator, clear the calculator's memory and use the following:

$$1 = P/Y$$

$$20000 = PV$$

$$13 = I$$

$$5 = N$$

$$\text{Solve for PMT} = \$5,686.29$$

Excel Financial Calculator (LT12)	
The Payment is \$5,686.29	
Present Value = PV	\$20,000.00
Years/Periods* = N	5.00
Payments/Year = P/Yr	1
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I _{EFF}	13.000%
Ann. Nom. Rate = I _{CON}	13.000%
Ann. Inflation = I _{OS}	
1 Period Rate =	13.00%
Future Value = FV	
Payments = PMT	\$5,686.29
Type = Type	
Payments at: End = 0, Begin = 1	

Case Study 4

Data

Kaili has reviewed the impact of inflation in the late 1970s. She reviewed one of her parent's investments during that time period and discovered that inflation was 20 percent and that her parent's investment made a 30 percent return.

Calculations

What was her parent's real return on this investment during that period?

Case Study 4 Answers

The traditional (and incorrect) method for calculating real returns is

Nominal return – inflation = real return. This formula would give you a real return of 10%: 30% – 20% = 10%.

The correct method is $(1 + \text{nominal return}) / (1 + \text{inflation}) - 1 = \text{real return}$

$$(1.30 / 1.20) - 1 = 8.33\%.$$

Excel Financial Calculator (LT12)	
Present Value = PV	
Years/Periods* = N	
Payments/Year = P/Yr	
<small>(Compounding: Annual = 1, Monthly = 12, Quarterly = 4)</small>	
Annual Interest = I_{EAR}	8.333%
Ann. Nom. Rate = I_{NOM}	30.000%
Ann. Inflation = I_{INF}	20.000%
Future Value = FV	
Payments = PMT	
Type = Type	
<small>Payments at: End = 0, Begin = 1</small>	

In this example, the traditional method overstates return by 20 percent ($(10\% / 8.33\%) - 1$). Be very careful of inflation, especially high inflation!